3 SEM TDC GEMT (CBCS)
GE 3 (A/B/C)

2024

(Nov/Dec)

MATHEMATICS

(Generic Elective)

Paper : GE–3

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : GE–3A

(Real Analysis)

1. *(a)* Define denumerable set. 1

(b) Show that the set I of all integers is countable. 3

(c) Find the supremum and infimum of the set

$$X = \{x \in I / x^2 \leq 49\}$$ 4

(d) State and prove the Archimedean property for real numbers. 4

Or

Show that R is a complete ordered field.

(e) If $S = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$, prove that $\inf S = 0$. 4

(f) State and prove Bolzano-Weierstrass theorem for sets. 4

2. (a) Define limit of a sequence. 1

(b) Prove that every bounded sequence has a convergent subsequence. 3

(c) Prove that every convergent sequence is bounded. 4

(d) Define Cauchy sequence. Prove that every sequence is convergent if and only if it is Cauchy sequence. 1+4=5

(e) Prove that the sequence $\left\{ x_n = \frac{1}{n} \right\}$ is a Cauchy sequence. 3

Or

Prove that the sequence $\left\{ x_n = \frac{n+1}{n} \right\}$ is a Cauchy sequence.

(f) Show that the sequence $\left\{ \frac{3n+7}{4n+8} \right\}$ is monotonic. Check whether it is monotonically increasing or decreasing. 3+1=4

Or

State and prove nested interval theorem for sequence. 4

3. (a) Define infinite series. 1

(b) Test the convergence of the series

$$1 + 2 + 3 + \ldots$$ 2

(c) Prove that the necessary condition for convergence of an infinite series $\Sigma\, u_n$ is

$$\lim_{n \to \infty} u_n = 0$$ 5

Or

Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

(d) Show that the series whose nth term is $\sin \frac{1}{n}$ is divergent. 2

Or

State Cauchy's root test.

(e) Test the convergence of any *two* of the following : 5×2=10

(i) $\sum \frac{1}{n^3}\left(\frac{n+2}{n+3}\right)^n$

(ii) $\sum \frac{1}{(\log n)^n}$

(iii) $\frac{1^3}{3}+\frac{2^3}{3^2}+\frac{3^3}{3^3}+\ldots+\frac{x^3}{3^n}+\ldots$

(iv) $\frac{1}{1+2}+\frac{2}{1+2^2}+\frac{3}{1+2^3}+\ldots$

4. (a) Prove that a sequence $\{f_n(x)\}$ on the set X is uniformly convergent if and only if for any given $\varepsilon > 0 \; \exists \, n_0 \in \mathbb{N}$ such that

$$n \geq n_0 \Rightarrow |f_{n+p}(x) - f_n(x)| < \varepsilon, \quad p \in \mathbb{N}$$ 5

Or

Show that the sequence $\{f_n\}$, where

$$f_n(x) = \frac{x}{1+nx^2}$$

is uniformly convergent on any closed interval I.

(b) Show that the series

$$\sum_{n=1}^{\infty} u_n(x)$$

is uniformly convergent on X if $\sum M_n$ is convergent series of positive constant such that $|u_n(x)| \leq M_n \forall n$ and $\forall \, x \in X$. 5

Or

Show that the series

$$\sum_{n=1}^{\infty} \frac{a_n x^n}{1+x^{2n}}$$

is uniformly convergent for all real x if

$$\sum_{n=1}^{\infty} a_n$$

is absolutely convergent.

(c) Define power series and give an example. 1

(d) If the power series

$$\sum_{n=1}^{\infty} a_n z^n$$

converges for particular value of z_0 of z, then show that it converges absolutely for every z for which $|z| < |z_0|$. 5

(e) Find the radius of convergence of the following series (any *one*) : 4

(i) $\sum (4+3i)^n z^n$

(ii) $\frac{z}{2} + \frac{1\cdot 3}{2\cdot 5} z^2 + \frac{1\cdot 3\cdot 5}{2\cdot 5\cdot 8} z^3 + \ldots$

(*Continued*)

Paper : GE–3B

(Cryptography and Network Security)

1. *(a)* What is cryptography? 2

(b) Briefly explain encryption and decryption with suitable examples. 2+2=4

(c) Define plain text and ciphertext with suitable examples. 2+2=4

(d) What are hashing and message digest? 2+2=4

(e) What is Caesar cipher? 2

2. Compare private key encryption and public key encryption with suitable examples. 3+3=6

3. What is digital signature? 2

4. Briefly explain the following network attacks (any *two*) : 4×2=8

(a) IP spoofing

(b) DDoS attack

(c) Teardrop attack

(d) Smurf attack

(*Turn Over*)

5. (a) What is ICMP? What are different messages used in ICMP? 2+4=6

(b) What is VPN? How does it work? 2+2=4

(c) Name two IPsec protocols. 2

(d) What is SYN flood? 2

(e) What is tunneling? 2

6. Briefly explain SNMPv1 architecture. 5

7. What is firewall? How can it be used to protect network? 2+4=6

8. Briefly explain the functionalities of Secure Electronic Transactions (SET). 6

9. Briefly explain the working principles of three different types of firewalls. 4×3=12

10. What is SSL (Secure Socket Layer)? 3

Paper : GE–3C

(Information Security)

1. What is information security? Briefly explain the three aspects/principles of information security. 2+6=8

2. Name five different types of information security. 5

3. What is Orange Book in information security? 3

4. Briefly explain the following information security threats (any *five*) : 3×5=15

(a) Virus

(b) Worms

(c) Bots

(d) Trojan horse

(e) Spyware

(f) Ransomware

5. Why is communication privacy important? 3

6. What is buffer overflow? 2

7. Briefly explain encryption and decryption with suitable examples. 3+3=6

8. What are plain text and ciphertext in cryptography? 2

9. How does symmetric-key encryption differ from asymmetric-key encryption? 4

10. What are hash function and message digest? 2+2=4

11. What is digital signature? Briefly explain the working principle of digital signature. 2+8=10

12. What is non-repudiation in digital signature? Briefly explain with suitable example. 2+2=4

13. Mention two uses of digital signature. 2

(*Continued*)

14. Briefly explain any *three* of the following : 4×3=12

(a) Intrusion detection

(b) Auditing

(c) Logging

(d) Tripwire

(e) System call monitoring

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