6 SEM TDC DSE MTH (CBCS) 6 (H)

2024

(May)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–6

(Mathematical Methods)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Let $f(x)$ be a periodic function of period 2π. Write the value of $f(x)$ to which it converges at the end points $x = \pm\pi$. 1

(b) Define Fourier series of a function $f(x)$ in the interval $(a, a+2\pi)$, $a \in R^+$. 2

(c) Find a Fourier series for the function $f(x) = x + x^2$ in the interval $(-\pi, \pi)$. 7

Or

Find the half range cosine and sine series of the function $f(x) = x$ in the interval $(0, \pi)$.

2. (a) Write the value of (i) $L\{2\}$ and (ii) $L\{\sin x\}$. 1+1=2

(b) Find (i) $L\{1-2\sin^2 x\}$ and (ii) $L\{t^2\}$. 2+2=4

(c) State and prove the second shifting theorem of Laplace transform. 4

Or

Find $L\{e^{-t}(2\sinh t + 7\cos 3t)\}$.

(d) Find $L\{t\cos 2t\}$. 4

Or

Find $L\{e^t \cos^2 2t\}$.

(e) Find the following (any *two*) : 3×2=6

(i) $L\{a+bt^2+c\sqrt{t}\}$

(ii) $L\{(\cos x+1)^2\}$

(iii) $L\{te^t \sin t\}$

3. (a) Write the value of $L^{-1}\left\{\frac{1}{s-2}\right\}$. 1

(b) Write the value of $L^{-1}\left\{\frac{1}{s^6}\right\}$. 1

(c) Define null function. 1

(d) Find the following (any *one*) : 3

(i) $L^{-1}\left\{\frac{3}{(s-3)^2+3^2}\right\}$

(ii) $L^{-1}\left\{\frac{1}{s^2-6s+10}\right\}$

(e) Find the following (any *one*) : 4

(i) $L^{-1}\left\{\frac{1}{(s^2+4)(s+1)^2}\right\}$

(ii) $L^{-1}\left\{\log\frac{s+7}{s+2}\right\}$

4. (a) Write the Fourier sine integral formula. 1

(b) Write the Dirichlet's conditions for Fourier transform. 2

(c) Find the Fourier sine transform of $f(x) = x$. 3

(d) State and prove the change of scale property of Fourier transform. 5

Or

Prove that

$$F\{x^n f(x)\} = (-i)^n \frac{d^n}{dp^n}[\bar{f}(p)]$$

(e) Answer the following (any *two*) : 7×2=14

(i) Find the Fourier transform of $f(x)$ defined by

$$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & x > 1 \end{cases}$$

(ii) Find the inverse Fourier transform $f(x)$ of $F(p) = e^{-|p|y}$.

(iii) Find the Fourier cosine and sine transforms of e^{-ax}, $a > 0$.

5. (a) Write the value of $L\left\{\frac{\partial y}{\partial x}\right\}$. 1

(b) Find $L\left\{\frac{\partial y}{\partial t}\right\}$. 2

(c) Solve using Laplace transform (any *two*) : 6×2=12

(i) $(D^2 + 4D + 5)y = 5$, $y(0)$, $y'(0) = 0$, $D \equiv \frac{d}{dt}$

(ii) $(D^2 + 3D + 2)y = e^{-t}$, $y(0) = 0$, $y'(0) = 1$

(iii) $(D^2 + 9)y = \sin t$, $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = 1$

(iv) $\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial t} + y$, $y(x, 0) = 6e^{-3x}$

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