Unit-2:

Classical Theory of Radiation

Introduction:

The word radiation is used in two senses: in one sense it is a process of transmission of energy from a hot body to a cold one in the form of waves and in the other sense it is used to indicate the energy that is transmitted. If the energy, on being absorbed by a body, heats it up, it is specially called thermal radiation. Any hot body emits thermal radiations. Thermal radiation is the transmission of heat energy which can occur in the absence of any material medium i.e., Radiation is a physical phenomenon that is the energy transferred without the active contact (Participation) of the intervening medium. Example: Heat from the sun reaches the earth after travelling through a very long distance in vacuum i.e. Transfers of energy in solar system, interstellar space and the galaxies.

Radiation of heat takes place along straight lines, like light, with the velocity of light. Radiant heat is, in fact, a form of electromagnetic waves, like light, but of much shorter wavelength. It suffers reflection, refraction, absorption and dispersion like light.

(In the process radiation, a medium is not required; however for conduction and convection, medium is required to flow of energy.)

Properties of Thermal Radiation:

1. Thermal radiation considered as electromagnetic radiation produced by the thermal motion of charged particles in matter.
2. All bodies above zero kelvins radiate thermal energy.
3. As this is electromagnetic wave, hence thermal radiation travels through vacuum with the velocity of light.
4. Thermal radiation travels in straight line and obeys the laws of reflection and refraction and also exhibits the phenomenon of interference, diffraction and polarization of light i.e. it has same nature as light. The only difference is that its average wavelength is greater than that of visible light. Therefore, the thermal radiation is called the Infra-red radiation.
5. Thermal radiations cannot be detected by human eyes or photographic plate but they can be detected by the instrument Bolometer.
6. Thermal radiation consists of quanta or photons of energy $h\nu$, here, h is the Plank's constant and ν is the frequency of radiation.
7. The quantity of heat radiated per second depends on the nature of the emitting surfaces, its surface area and its temperature.
8. The nature of the thermal radiation depends upon temperature. At a low temperature, a body emits radiations of long wave length.

   $$i.e \quad T \propto \frac{1}{\lambda}, \qquad \text{here, } T \text{ is the temperature of the body.}$$

9. In general the wavelength of thermal radiation in the range: $8000\overset{0}{A}$ to $0.4mm$
10. Exchange of heat between a body and its surrounding continues till a dynamic thermal equilibrium obtained.
11. The thermal radiation produces heat when they are absorbed by a body. Good absorbers are good radiator and vice-versa.
12. When thermal radiations fall on a body, they are partially reflected, absorbed and transmitted.

Some Definitions:

Reflectance: (r)

The ratio of the amount of thermal radiation reflected (R) by a body in a certain time to the total amount of incident thermal radiations (Q) falling upon the body in the same time is called reflectance i.e.

Reflectance, $r = \frac{R}{Q}$

Absorptance: (a)

The ratio of the amount of thermal radiation absorbed (A) by a body in a certain time to the total amount of thermal radiation (Q) incident upon the body in the same time is called absorptance i.e.

Absorptance, $a = \frac{A}{Q}$

Transmittance: (t)

The ratio of the amount of thermal radiations transmitted (T) by a body in a certain time to the total amount of thermal radiation incident (Q) on it in the same time is called transmittance i.e.

Transmittance, $t = \frac{T}{Q}$

The reflectance (r), absorptance (a) and transmittance (t) of a body depends on the

1. Nature of the surface of body.
2. The frequency of incident radiation.
3. Mathematically, $r + a + t = 1$ and $R + A + T = Q$.

Total energy density:

The total energy of radiations at any point is the total radiant energy per unit volume around that point for all the wavelengths taken together. This is generally expressed by 'u' and its unit is joule m^{-3}.

Spectral energy density:

The spectral energy density for a particular wavelength is the energy per unit volume per unit range of wavelength. This is denoted by u_λ.

Emissive power: (e)

The spectral emissive power of a body at a particular wavelength is the radiant energy emitted per unit time per unit surface area of the body within a unit wavelength range. The emissive power of a body corresponding to a wavelength λ is e_λ.

Emissivity:

It is defined as the ratio of emissive power of a given body (e) to that of the perfectly black body (E). Emissivity value depends upon the nature of body which emitting radiations.

Absorptive power or Absorptance: (a)

The absorptive power of a body at a particular temperature and for a particular wavelength is defined as the ratio of the radiant energy absorbed per unit surface area per unit time to the total energy incident on the same area of the body in unit time within a unit wavelength range. The absoptive power of a body corresponding to a wavelength λ is a_λ.

Blackbody Radiation:

A body which absorbs all the incident radiations and does not reflects and nor transmits any of the incident radiation is called black body i.e. for black body; reflectance $r = 0$, transmittance $t = 0$ and absorptance $Q = 1$.

In practices no substance possesses strictly the properties of a black body. Lamp black and the Platinum black are the nearest approach to a black body. Black bodies in practice are:

i. **Fery's black body:**

A theoretical model given by Fery is shown in fig.1. It is a hollow double walled metal sphere, in which inner surface coated with Lamp black and the outer surface Polished with Nickel. Therefore a radiation which enters by small opening O, all radiations are wholly absorbed due to multiple reflections. *(Lamp black or Carbon black is the material which is produced by incomplete combustion of petroleum products like Fossil fuel (wood, coal), crude oil (unrefined petroleum), Coal tar (coal gas) etc. This lamp black can absorb around 96% to 98% of radiation falling on it and considered as black body.)*

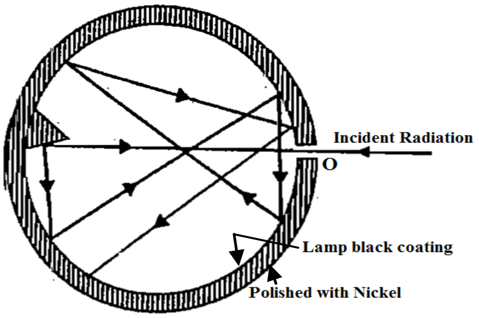
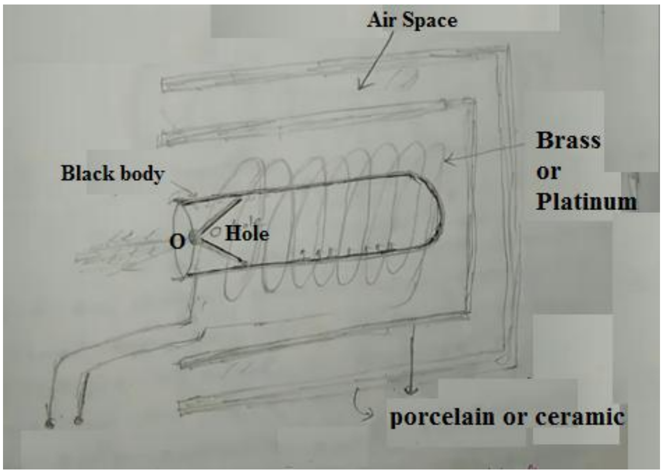


Fig.1: Fery Black Body



ii. Wien's black body:

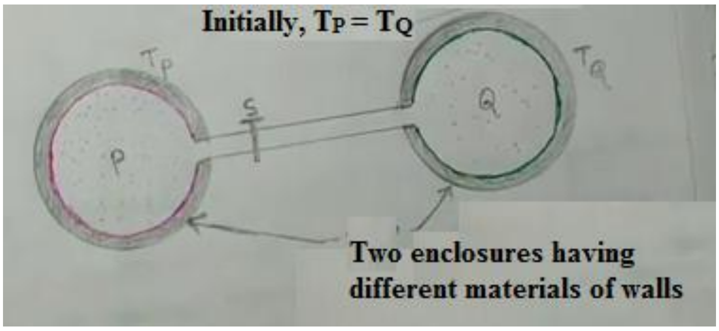
Wilhelm Wien is a German Physicist designed a cylindrical type of black body which is consists by a hollow cylinder made of brass or platinum having coil of thin platinum wire wound over it. The inner surface of the cylinder is painted with lamp black. A co-axial porcelain or ceramic material tube with air space protects the cylinder. When current passed through the coil, the cylinder gets heated the inner surface of the cylinder. A small hole "O" has been designed that the radiation from cylinder through black surface will be emitted through O only. Here it is assumed that through the small hole O, radiations emitted having all the frequencies.

Ideal black body: A black body which absorbs all incident radiations and emits all frequencies of radiations is called ideal black body. An ideal black body not is black in color.

Pure Temperature dependence:

According to pure temperature dependence of the radiation by materials, the quality and quantity of radiation depend only on the temperature at which the wall of the enclosure are maintained and does not depend on the nature of the walls of the enclosures.

To prove the above, let us consider two cavities / enclosures P and Q having walls of different materials heated to the same temperature and filled with radiant energy as shown in figure. The two enclosures are joined by a tube fitted with a key 'S', which is initially closed.



As the walls of two enclosures made up of two different materials, hence initially we assume that, the density of radiant energy (energy per unit volume) in the enclosure P is more than that of Q. Now if the key S is open then radiation will flow from P to Q till equilibrium is attained. When the radiation is received by enclosure Q, the temperature of Q will be increased, as the result, a temperature difference will be developed between the two enclosures i.e. we found $T_Q > T_P$ but initially both are at same temperature.

This result contradicts with 2nd law of thermodynamics that Q can be used as a sources and P as a sink in a heat engine and work can be obtained without any external agent /energy, which is not to be acceptable.

Therefore our initial assumption that the radiation density in P is greater than Q is false. Hence, the energy density of the radiation in a uniform temperature enclosure depends on its temperature only and does not depend on the nature of walls of the enclosures.

Kirchhoff's Law:

This law states that the ratio of the emissive power e_λ to the absorptive power a_λ for a given wavelength at a given temperature is the same for all bodies i.e. constant and equal to the emissive power E_λ of a perfectly black body at that temperature, i.e.

$$\frac{e_\lambda}{a_\lambda} = E_\lambda \text{(Constant)}$$

Proof:

Let us consider a body placed in an isothermal enclosure. Let dQ be the amount of radiant energy of wavelength lying between λ and $\lambda + d\lambda$ incident on unit surface area per second as shown in figure. If a_λ is the absorptive power of the body for the wavelength λ then the amount of radiant energy absorbed by unit surface area of the body per second will be $a_\lambda dQ$.

Therefore, the amount of thermal radiations reflected and transmitted per second per unit area will be

$$= dQ - a_\lambda dQ$$

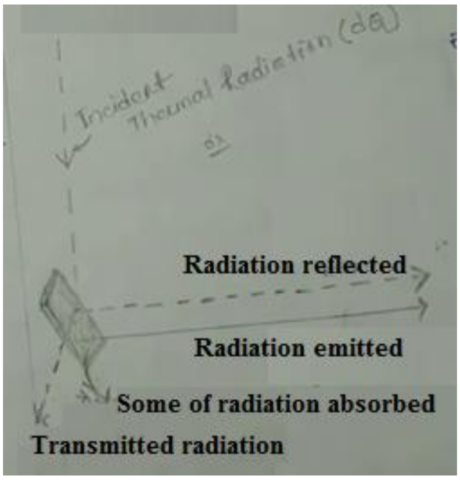
$$= (1 - a_\lambda) dQ \qquad \text{--- (1)}$$

We know that the body emits some amount of energy due to its temperature. If e_λ be the emissive power of the body corresponding to that wavelength, then the amount of thermal radiations emitted per second per unit area by the body $= e_\lambda d\lambda$.

Thus, total amount of thermal radiations going away from the body

$$= (1 - a_\lambda) dQ + e_\lambda d\lambda \qquad \text{--- (2)}$$

In accordance with the law of conservation of energy, the total thermal radiations going away from the body must be same as that incident on the body i.e.



$$dQ = (1 - a_\lambda) dQ + e_\lambda d\lambda \qquad \text{--- (3)}$$

$$\Rightarrow dQ = dQ - a_\lambda dQ + e_\lambda d\lambda$$

$$\Rightarrow e_\lambda d\lambda = a_\lambda dQ \qquad \text{--- (4)}$$

But for a perfect black body; $a_\lambda = 1$ and $e_\lambda = E_\lambda$

Therefore, from equation (4), we get

$$dQ = E_\lambda d\lambda \qquad \text{--- (5)}$$

Now, substituting the value of dQ in equation (4), we get

$$\Rightarrow e_\lambda d\lambda = a_\lambda E_\lambda d\lambda$$

$$\Rightarrow \frac{e_\lambda}{a_\lambda} = E_\lambda \text{(Constant)} \qquad \text{--- (6)}$$

i.e. the ratio of the emissive power to the absorptive power is equal to emissive power of a perfectly black body at a given temperature, which is the Kirchhoff's law.

Importance of Kirchhoff's law:

Kirchhoff's law shows that if emissive power e_λ is large then absorptive power a_λ is also large as E_λ is constant. This means that good emitters are good absorbers.

Application of Kirchhoff's law:

Kirchhoff's

1. When a green glass heated in a furnace with high temperature and when taken out in dark room it glows with red light. Green glass is a good absorber of red light and a good reflector of green light. On heating green glass becomes a good emitter of red light as per Kirchhoff's law.
2. Let a polished metal with a dark spot on it be heated to high temperature and taken into a dark room. The spot begins to shine brightly indicating that good absorbers are good emitters.
3. When a decorated china pottery (cup) heated in 1000^0c and then taken out the decorated pottery and taken into a dark room then it appears brighter than white china pottery because decorations are better absorbers and better emitters.

Radiation Pressure:

Radiation pressure is defined as the pressure exerted upon any surface when it exposed to electromagnetic radiation. James Clerk Maxwell, a Scottish Physicist was 1st explained the radiation pressure by interaction of electromagnetic waves or particles called photons. Therefore all electromagnetic radiations exert a definite pressure on the surface on which it is incident.

According to the quantum theory put by Max Planck, a German Physicist, a radiation of frequency ν consists of photons of energy $h\nu$, where h is Planck's constant. As per the theory of relativity (by Albert Einstein), mass and energy are mutually convertible where the energy of mass m is mc^2, c is the velocity of light.

Hence,

$$h\nu = mc^2$$

$$\Rightarrow m = \frac{h\nu}{c^2}$$

Now the momentum of photon $= mass \times velocity$

$$= \frac{h\nu}{c^2} \times c$$

$$= \frac{h\nu}{c}$$

When n numbers of such photons forming the radiation are incident on a unit area then the total momentum imparted or exerted per unit time per unit area is

$$= n\frac{h\nu}{c} \qquad \text{---- (1)}$$

But we know that, the pressure P is defined as force per unit area and is numerically equal to the total momentum imparted per unit time per unit area

Hence,

$$\text{Pressure, } P = \frac{nh\nu}{c} \qquad \text{--- (2)} \quad \left[P = \frac{F}{A} = m\times\frac{dv}{dt}\times\frac{1}{A} = m\times\frac{v}{t}\times\frac{1}{A} = mv\times\frac{1}{t}\times\frac{1}{A}\right]$$

The quantity $nh\nu$ represents the total energy incident on the surface per unit area per unit time.

Let, $E = nh\nu$

$$\therefore \quad P = \frac{E}{c} \qquad \text{--- (3)}$$

If u be the energy density i.e. energy per unit volume in vacuum, then total energy passing through any area A (imaginary area in vacuum) of the surface normal to the radiation per unit time

$$= u \times A \times c \qquad [\because \text{In unit time radiation travels the distance c}]$$

So, the total energy radiations which incident per unit area per unit time is

$$E = \frac{u \times A \times c}{A} = uc \qquad \text{--- (4)}$$

Substituting (4) in (3), we found

$$\therefore \quad P = \frac{uc}{c} = u \text{ (Energy density)} \qquad \text{--- (5)}$$

Hence for normal incidence on a surface, the pressure of radiation is equal to energy density.

Pressure of diffuse radiation:

When the beams or photons contained in the radiations, move in all possible directions then that radiations are known as diffuse radiation.

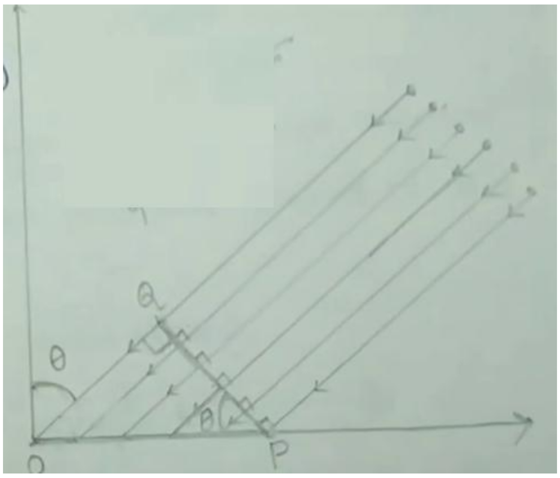
Let a diffused radiation be incident on a surface OP at an angle θ with the normal as shown in fig.1. Surface PQ (imaginary surface in vacuum) is taken to be normal to the incident beam such that the total energy of radiation incident on PQ per unit time is

$$= Energy \times Surface\ area$$

$$= uc \times A \qquad \text{--- (6)}$$

Here, A is the surface area of PQ.

In accordance with the principle of conservation energy, the total energy of radiation crossing the surface PQ is equal to the energy of radiation incident on the surface PQ per unit time. From fig.1, we have in ΔOPQ



$$\frac{PQ}{OP} = \cos\theta$$

$$\Rightarrow \frac{A}{A'} = \cos\theta$$

Where A' is the total surface area of surface OP.

But, the energy of radiation incident on the surface OP per unit area per unit time is

$$= \frac{ucA}{A'}$$

$$= \frac{ucA'\cos\theta}{A'}$$

$$= uc\cos\theta \qquad \text{--- (7)}$$

The net momentum incident on the surface OP per unit area per unit time will be

$$= \frac{uc\cos\theta}{c} \qquad \because \quad P = \frac{E}{c}$$

$$= u\cos\theta \qquad \text{--- (8)}$$

If we consider the horizontal component of momentum along the direction normal to OP (downward) then it will be

$$= (u\cos\theta)\cos\theta$$

$$= u\cos^2\theta \qquad \text{--- (9)}$$

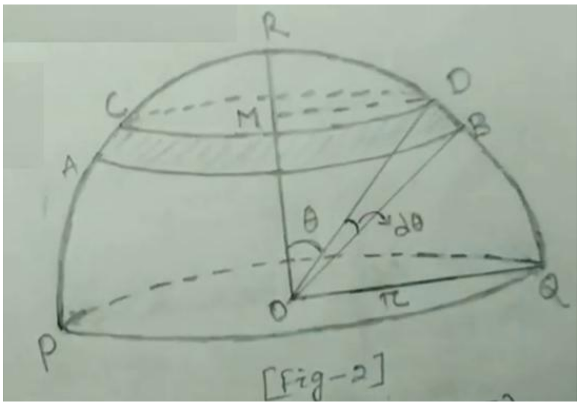
If all the momentum is absorbed then the total rate of change of momentum per unit time per unit area normal to the surface OP gives the pressure of diffuse radiation.

$$\therefore \quad Pressure \quad P = u\cos^2\theta \qquad \text{--- (10)}$$

The radiation is incident from all possible directions with equal probability for diffuse radiation. The average value of $\cos^2\theta$ over all directions will give the total pressure of diffuse radiation. To calculate average value of $\cos^2\theta$, we consider the radiation to be equivalent to a large number of beams or photons (say N) of equal intensity distributed evenly (uniformly) in all directions.

Let us consider an imaginary hemisphere of radius r with center at point O as shown in fig.2. Cut out a ring shape element ABCD using two cones of angles θ and $\theta + d\theta$ drawn from point O.

From ΔODM, $\sin\theta = \frac{MD}{OD} = \frac{MD}{r} \Rightarrow MD = r\sin\theta$ and $DB = rd\theta$



Therefore, area of the elementary ring ABCD,

$$= 2\pi \times radius\ of\ ring\ MD \times thickness\ DB$$

$$= 2\pi \times r\sin\theta \times rd\theta$$

$$= 2\pi r^2 \sin\theta d\theta \qquad \text{--- (11)}$$

Since the N numbers of photons are uniformly distributed over the surface of the hemisphere of surface area $2\pi r^2$.

Therefore, the small number of photons/beams which passing through the surface area $2\pi r^2 \sin\theta d\theta$ of ring will be

$$dN = \frac{N}{2\pi r^2} \times 2\pi r^2 \sin\theta d\theta$$

$$\Rightarrow dN = N\sin\theta\, d\theta \qquad \text{--- (12)}$$

$$\Rightarrow \frac{dN}{N} = \sin\theta\, d\theta \qquad \text{--- (13)}$$

This represents the fraction of beams incident between the angle θ and $\theta + d\theta$.

Since, For the surface area $2\pi r^2$, the number of photons							= N
,,	,,	,,	1,	,,	,,	,,	$= \frac{N}{2\pi r^2}$
,,	,,	,,	$2\pi r^2 \sin\theta d\theta$		,,	,,	$= \frac{N}{2\pi r^2} \times 2\pi r^2 \sin\theta d\theta$

Therefore small change in pressure dP will be

$$dP = \frac{dN}{N} \times u\cos^2\theta$$

$$\Rightarrow dP = \sin\theta\, d\theta \times u\cos^2\theta \qquad \text{--- (14)}$$

Since, For N particle, change in pressure			$= u\cos^2\theta$
,, 1	,,	,,	$= \frac{u\cos^2\theta}{N}$
,, dN	,,	,,	$= \frac{u\cos^2\theta}{N} dN$

Integrating both side of equation (14), we get the net pressure exerted by all possible of rings within the hemisphere when θ varies from 0 to $\frac{\pi}{2}$.

Hence,

$$\int_0^P dP = u\int_o^{\pi/2} \cos^2\theta \sin\theta\, d\theta$$

Let $\cos\theta = x \Rightarrow -\sin\theta\, d\theta = dx$ and limit changes, 1 to 0.

$$\therefore P = u\int_1^0 x^2(-dx)$$

$$\Rightarrow P = u\int_0^1 x^2 dx$$

$$\Rightarrow P = u\left[\frac{x^3}{3}\right]_0^1$$

$$\Rightarrow P = \frac{u}{3}$$

Here we concluded that, the pressure of diffused radiation is equal to one third of its energy density. (But for normal incidence, $P = u$)

Stefan Boltzmann Law:

Josef Stefan an Austrian physicist gave Stefan's law in 1879 which states that "the amount of total energy of thermal radiation emitted per unit time per unit area of a perfectly black body is directly proportional to the forth power of the temperature of the body."

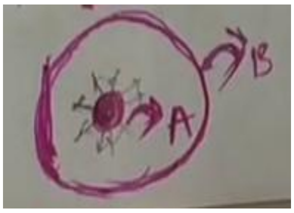
Mathematically,

$$E \propto T^4$$
$$\Rightarrow E = \sigma T^4$$

Where, σ is a constant of proportionality called Stefan's constant and its value is $5.67\times10^{-8}\, watt\, m^{-2}\, K^{-4}$ or $Joule\, m^{-2}\sec^{-1} K^{-4}$.

Later on in 1884, Stefan's students Ludwig Boltzmann (also an Austrian physicist) proved this law theoretically and this law came to be known as Stefan's – Boltzmann law. Stefan considered only emission and not to be the net loss but Boltzmann considered net loss by the black body.

Explanation: Let a perfect black body A at absolute temperature T is kept inside a black body B which is at absolute temperature T_0 .



Then, amount of heat lost / emitted by black body A is

$$= \sigma T^4$$

And, amount of heat absorbed by black body A from black body B is

$$= \sigma T_0^{\,4}$$

So, net amount of heat lost / emitted the black body A per unit time per unit area is

$$= \sigma\left(T^4 - T_0^{\,4}\right)$$

This is known as Stefan Boltzmann law.

Thermodynamic Proof:

In our previous section we have found that heat radiation (either incident normally or diffused) exerts pressure on the surface. Since the black body radiations/photons exert pressure and possess energy, therefore they behave like a gas molecule (i.e. gas molecules enclosed in a vessel exerts pressure on the wall of the vessel) as shown in fig. Thus the laws of thermodynamics and Maxwell's thermodynamic relations can be applied to a black body radiation just as in case of gas. So just imagine radiation/photons are enclosed in vessel as like gas.

Now, the total energy U of the radiation is given by

$$U = uV$$

And pressure, $P = \frac{1}{3}u$

Where, u is the energy density of radiation inside a uniform temperature enclosure at temperature T and V is the volume.

According to Maxwell's 2nd thermodynamic relation, we know that

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \qquad \text{--- (1)}$$

Where, S is the entropy.

Multiplying both sides by T with equation (1), we get

$$\left(\frac{T\partial S}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V$$

$$\Rightarrow \left(\frac{\partial Q}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V \qquad \because \quad T\partial S = \partial Q$$

Now, according to 1st law of thermodynamics, we know that

$$\partial Q = \partial U + P\partial V \qquad \text{--- (2)}$$

Putting the value of ∂Q in above equation, we get

$$\left(\frac{\partial U + P\partial V}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V$$

$$\Rightarrow \left(\frac{\partial U}{\partial V}\right)_T + \left(\frac{P\partial V}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V$$

$$\Rightarrow \left(\frac{\partial U}{\partial V}\right)_T + P = T\left(\frac{\partial P}{\partial T}\right)_V$$

$$\Rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P \qquad \text{--- (3)}$$

But, $U = uV \Rightarrow \frac{\partial U}{\partial V} = u$ And $P = \frac{1}{3}u \Rightarrow \frac{\partial P}{\partial T} = \frac{1}{3}\frac{\partial u}{\partial T}$, hence the equation (3) will be

$$u = T\left(\frac{1}{3}\frac{\partial u}{\partial T}\right)_V - \frac{1}{3}u$$

$$\Rightarrow u + \frac{1}{3}u = \frac{1}{3}T\left(\frac{\partial u}{\partial T}\right)_V$$

$$\Rightarrow \frac{4}{3}u = \frac{1}{3}T\left(\frac{\partial u}{\partial T}\right)_V$$

$$\Rightarrow 4u = T\left(\frac{\partial u}{\partial T}\right)$$

$$\Rightarrow 4u\partial T = T\partial u$$

$$\Rightarrow \frac{\partial u}{u} = 4\frac{\partial T}{T}$$

Integrating both side of the above equation, we get

$$\int\frac{\partial u}{u} = 4\int\frac{\partial T}{T}$$

$$\Rightarrow \ln u = 4\ln T + \ln a$$

Here, $\ln a$ is the integration constant.

$$\Rightarrow \ln u = \ln T^4 + \ln a$$

$$\Rightarrow u = a\,T^4 \qquad \text{--- (4)}$$

Multiplying by the velocity of light C on both sides of equation (4), we get

$$\Rightarrow uc = ca\,T^4$$

$$\Rightarrow E = \sigma T^4$$

Here, $ca = \sigma$ is called Stefan's constant.

Wien's Displacement Law:

Wilhelm Wien a German physicist derived Wien's displacement law in 1893 which states that "the product of wavelength λ_m corresponding to the maximum energy and the absolute temperature T is always constant for a black body/anybody radiation" i.e.

$$\lambda_m T = Constant(b)$$

Here the constant (b) is known as Wien's displacement constant and has a fixed value $\approx 0.2896 \times 10^{-2}\, meter\ Kelvin$.

Thus radiation of a particular wavelength at a certain temperature is adiabatically altered to another wavelength then the temperature changes in the inverse ratio. This is the usual statement of Wien's displacement law i.e. the temperature is inversely proportional to a particular wavelength.

Proof: (Deduction uncompleted- Very lengthy)

To prove this law, imagine a spherical body (enclosure) of perfectly reflecting walls and having tendency to expand due to radiation pressure as shown in fig.1. Because this body (enclosure) is filled with diffuse radiation of energy density u at a uniform temperature T.

The total internal energy U of diffuse radiation at volume V of the body (enclosure) will be

$$U = uV \qquad \text{-- (1)}$$

The total heat is in the body is constant because process/situation being considered as adiabatic i.e. $dQ = 0$, where Q is the total heat radiation and inside the body it is constant.

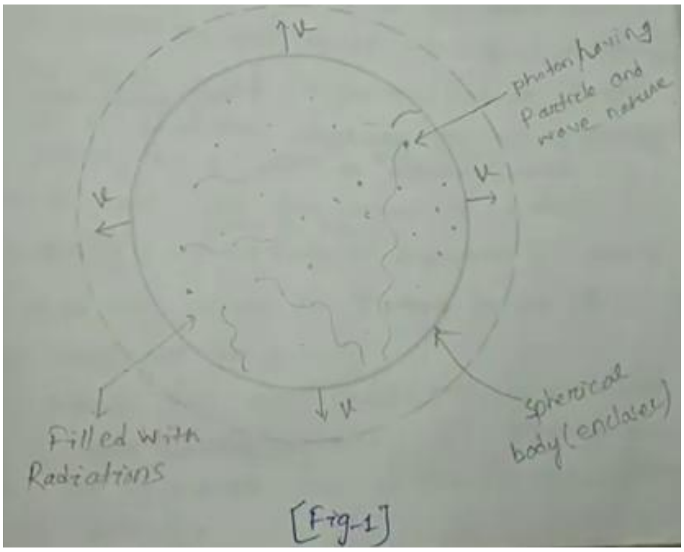
Let v be the uniform velocity with which the body/enclosure expanding adiabatically as shown in fig.1. According to the 1st law of thermodynamics

$$dQ = dU + dW \qquad \text{--- (2)}$$

Where, $dW = PdV =$ work done, P is the pressure of diffuse radiation which is one third of the energy density i.e. $P = \frac{u}{3}$. Putting the values of P and U in equation (2), we get

$$dQ = dU + dW$$

$$\Rightarrow 0 = d(uV) + PdV \qquad \because \text{ For adiabatic expansion, } dQ = 0$$



$$\Rightarrow 0 = udV + Vdu + \frac{u}{3}dV$$

$$\Rightarrow 0 = \frac{4}{3}udV + Vdu$$

$$\Rightarrow \frac{4}{3}udV = -Vdu$$

$$\Rightarrow \frac{4}{3}\frac{dV}{V} = -\frac{du}{u}$$

Integrating both side of above equation, we get

$$\Rightarrow \frac{4}{3}\int\frac{dV}{V} = -\int\frac{du}{u}$$

$$\Rightarrow \frac{4}{3}\ln V = -\ln u + \text{Constant}$$

$$\Rightarrow \ln V^{\frac{4}{3}} + \ln u = \text{Constant}$$

$$\Rightarrow \ln\left(V^{\frac{4}{3}}u\right) = \text{Constant}$$

Now from Stefan's law, we know that

$u = aT^4$, where a is a constant.

$$\therefore \quad \ln\left(V^{\frac{4}{3}}aT^4\right) = \text{Constant}$$

$$\Rightarrow V^{\frac{4}{3}}aT^4 = \text{Constant}$$

$$\Rightarrow \left(V^{\frac{1}{3}}T^4\right)^4 = \text{Constant}$$

$$\Rightarrow V^{\frac{1}{3}}T = \text{Constant} \qquad \text{--- (3)}$$

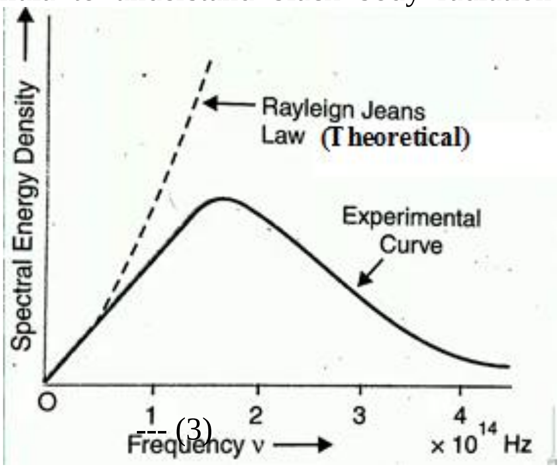
Relation between Wavelength λ and volume V: (Deduction uncompleted- Very lengthy)

$$\frac{dV}{V} = 3\frac{d\lambda}{\lambda} \qquad \textbf{(Wien's displacement law- Part -2)}$$

Ultraviolet Catastrophe:

The ultraviolet catastrophe is also called Rayleigh – Jeans catastrophe (Paradox i.e. confusion). It predicts that a perfect blackbody emits thermal radiation in all frequency range.

The emission of energy density will increases with increase of frequency or increase with decrease of wave length (what we discussed previously). This has been predicted / estimated by considering the two Rayleigh – Jeans equations. Rayleigh – Jeans derive a formula to understand black body radiation spectrum. Energy density for wavelength range λ and $\lambda + d\lambda$ is



$$u_\lambda d\lambda = \frac{8\pi\, k_B T}{\lambda^4} d\lambda \qquad \text{--- (1)}$$

$$\because \quad \lambda = \frac{c}{\nu} \Rightarrow d\lambda = \left|\frac{c}{\nu^2} d\nu\right|$$

$$\therefore \quad u_\nu d\nu = \frac{8\pi\nu^4}{c^4}\frac{c}{\nu^2} d\nu k_B T = \frac{8\pi\nu^2 d\nu}{c^3} k_B T \qquad \text{--- (2)}$$

The total energy density in terms of frequency is

$$U_\nu = \int_0^\infty u_\nu d\nu = \int_0^\infty \frac{8\pi\, k_B T \nu^2 d\nu}{c^3} = \frac{8\pi\, k_B T}{c^3}\int_0^\infty \nu^2 d\nu \qquad \text{--- (3)}$$

From both the graph it can be found that experimental and theoretical curves shows similar result only at low frequencies (high wavelength). But at high frequencies (low wavelength) theoretical curve shows that energy density increases to infinity whereas the experimental curve show after attaining the maximum value the energy density decreases. With further increase in frequency or decrease in wavelength, the energy density $U_\nu or\ U_\lambda$ decreases. This decrease of $U_\nu or\ U_\lambda$ happens in ultraviolet region only (even temperature increases to higher to higher). This paradox is called ultraviolet catastrophe (paradox).

i.e. at low frequency or higher wavelength Rayleigh-Jeans law agree with experimental black body curve, but at high frequency or low wavelength this law is not agree with black body curve. In violet colour wavelength region, according to this law (equation-3) energy density should be so large, but experimentally it is not found this like. This is called ultraviolet catastrophe.

(*It is observed that Wien's 5th power law agrees with experimental result only in lower wavelength region, while Rayleigh – Jeans law for distribution of energy of higher wavelength region and fail completely for shorter wavelength. This result is called ultraviolet catastrophe or Rayleigh – Jeans catastrophe (Paradox).)*