DIGITAL CIRCUITS

DIGITAL ELECTRONICS AND LOGIC GATES

Analogue and Digital signals:

Those electrical signals or wave forms in which electric current or voltage varies continuously with time are called analog signals or wave forms. Wave forms (Sine wave) shown in fig. 1 (a) is called analog signals. Electronic circuits in which continuous, time-varying signals are used are called analog circuit. Electronics circuits like amplifiers, oscillators, and rectifier are such analog circuits.

Electrical signals which have only two discrete levels of current or voltage (0 for Low and 1 for High) are called digital signals or digital wave form. Wave forms (Pulsating wave form) shown in fig. 1 (b) is called digital signals. Electronic circuits in which the current and voltage signals have only two levels (either ON or OFF) are called digital circuit. Logic gates are the digital circuits.

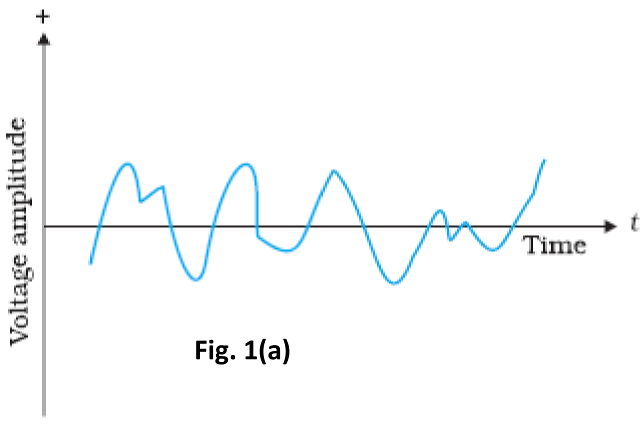


Fig. 1(a)

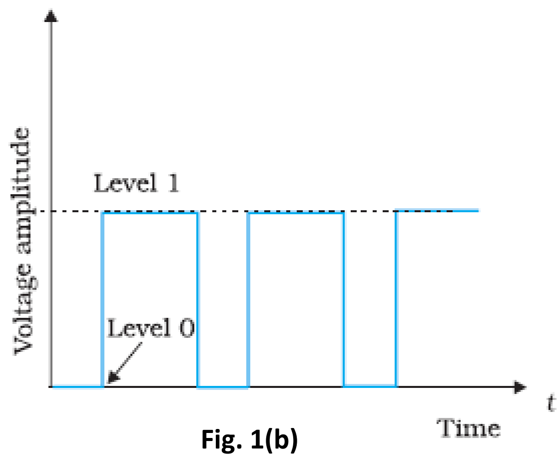


Fig. 1(b)

Logic gates:

A logic gate is an electronic circuit which performs logic functions or takes a logic decision. It has one or more inputs and only one output. These gates allow the signal to pass through only when some logical conditions are satisfied, so they are called logic gates.

Logic gates can be classified into two:

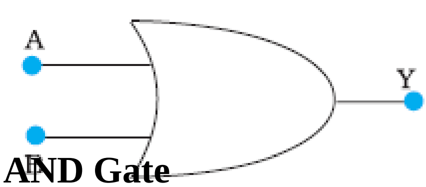
1. Fundamental gates
   (i) OR-Gate (ii) AND- Gate (iii) NOT- Gate
2. Derived gates:
   (i). NOR-Gate (ii) NAND- Gate (iii) EX-OR Gate (iv) EX-NOR Gate

In the derived gates, NAND and NOR gates are called as Universal Gates because any other logic gates can be constructed only by using either NAND or NOR gate.

OR Gate

An OR gate has two or more inputs with one output. The logic symbol and truth table are shown in figure. The output Y is 1 when either input A or input B or both are 1s, that is, if any of the input is high, the output is high. The Boolean equation (Logic equation) of OR gate is

$$Y = A + B$$



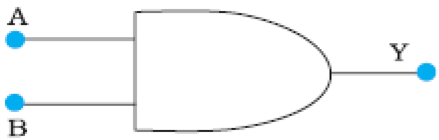
Truth Table

A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

AND Gate

An AND gate has two or more inputs and one output. The output Y of AND gate is 1 only when input A and input B are both 1. The logic symbol and truth table for this gate are given in figure. The Boolean equation (Logic equation) of AND gate is

$$Y = A \bullet B$$



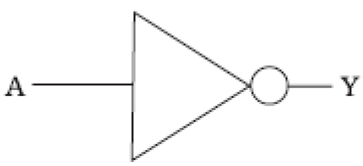
Truth Table

A	**B**	$Y = A \bullet B$
0	0	0
0	1	0
1	0	0
1	1	1

NOT gate

The NOT gate has only one input and one output. It produces a ‘1’ output if the input is ‘0’ and vice-versa. That is, it produces an inverted version of the input at its output. This is why it is also known as an inverter. The commonly used symbol together with the truth table for this gate is given in figure and the Boolean equation (Logic equation) of NOT gate is

$$Y = \overline{A}$$



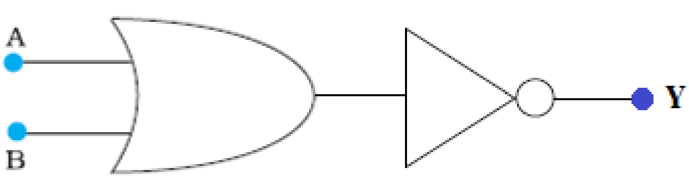
Truth Table

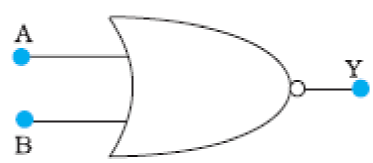
A	$Y = \overline{A}$
0	1
1	0

NOR gate

The logic gate in which the output of the OR gate is given to the input of NOT gate (as shown in fig. 1.a) is called the NOR gate. The logic symbol and truth table for this gate are given in fig. 1(b) and fig. 1(c) respectively. The Boolean equation (Logic equation) of OR gate is

$$Y = \overline{A + B}$$





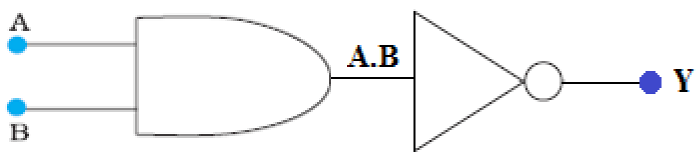
Truth Table

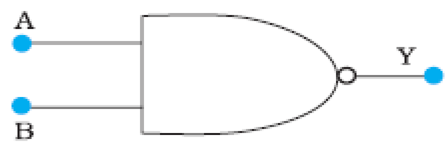
A	**B**	$Y = \overline{A + B}$
0	0	1
0	1	0
1	0	0
1	1	0

NAND gate

The logic gate in which the output of the AND gate is given to the input of NOT gate (as shown in fig. 1.a) is called the NAND gate. The logic symbol and truth table for this gate are given in fig. 1(b) and fig. 1(c) respectively. The Boolean equation (Logic equation) of NAND gate is

$$Y = \overline{A \bullet B}$$





Truth Table

A	B	$Y = \overline{A \bullet B}$
0	0	1
0	1	1
1	0	1
1	1	0

EX-OR gate

The logic gate in which the OR, AND and NOT gates are connected as shown in fig. 1. (a) is called the Exclusive OR gate (abbreviated as X-OR gate). The logic symbol and truth table for this gate are given in fig. 1(b) and fig. 1(c) respectively. The Boolean equation (Logic equation) of X-OR gate is

$$Y = A \bullet \overline{B} + \overline{A} \bullet B = A \oplus B$$

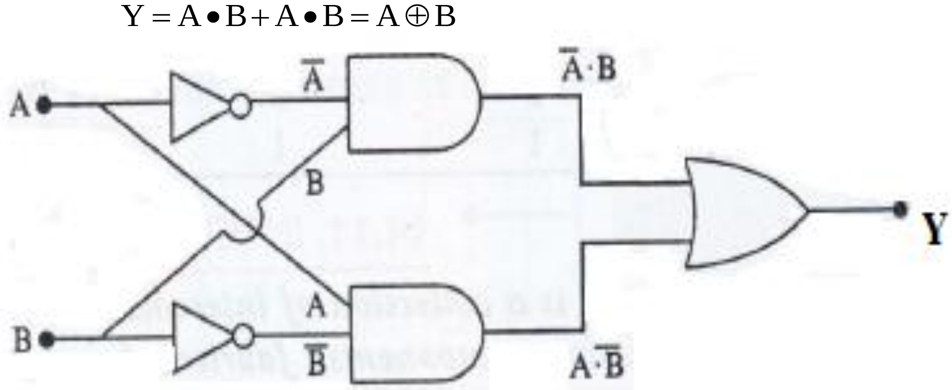


Fig. 1(a)

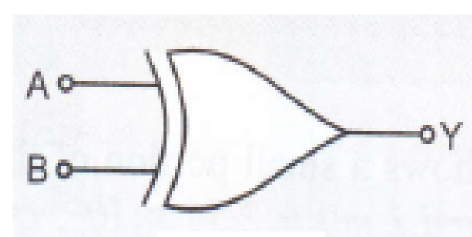


Fig. 1(b)

Truth Table

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \bullet B$	$A \bullet \overline{B}$	$Y = A \bullet \overline{B} + \overline{A} \bullet B = A \oplus B$
0	0	1	1	0	0	0
0	1	1	0	1	0	1
1	0	0	1	0	1	1
1	1	0	0	0	0	0

EX-NOR gate

The logic gate in which the output of the EX-OR gate is given to the input of NOT gate (as shown in fig. 1.a) is called the EX-NOR gate. The logic symbol and truth table for this gate are given in fig. 1(b) and fig. 1(c) respectively. The Boolean equation (Logic equation) of EX-NOR gate is

$$Y = \overline{A \oplus B}$$

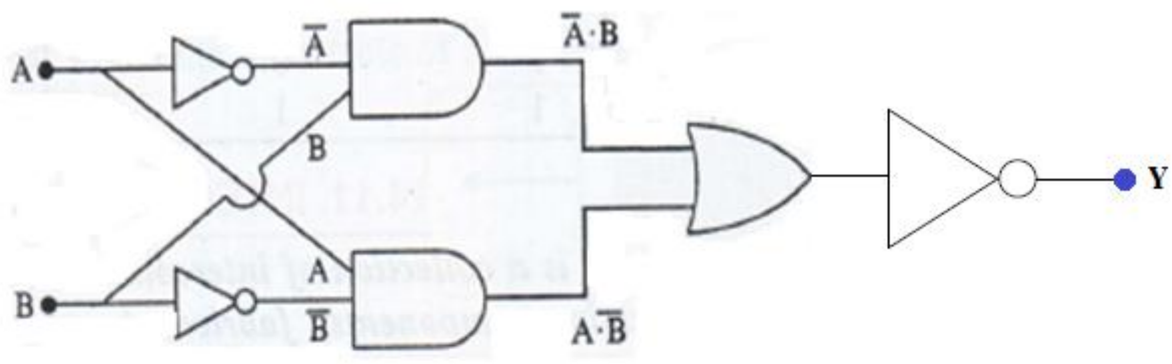


Fig. 1(a)

Truth Table

A	**B**	$\overline{A}$	$\overline{B}$	$\overline{A}\bullet B$	$A\bullet\overline{B}$	$Y = A\bullet\overline{B}+\overline{A}\bullet B = A\oplus B$	$Y=\overline{A\oplus B}$
0	0	1	1	0	0	0	1
0	1	1	0	1	0	1	0
1	0	0	1	0	1	1	0
1	1	0	0	0	0	0	1

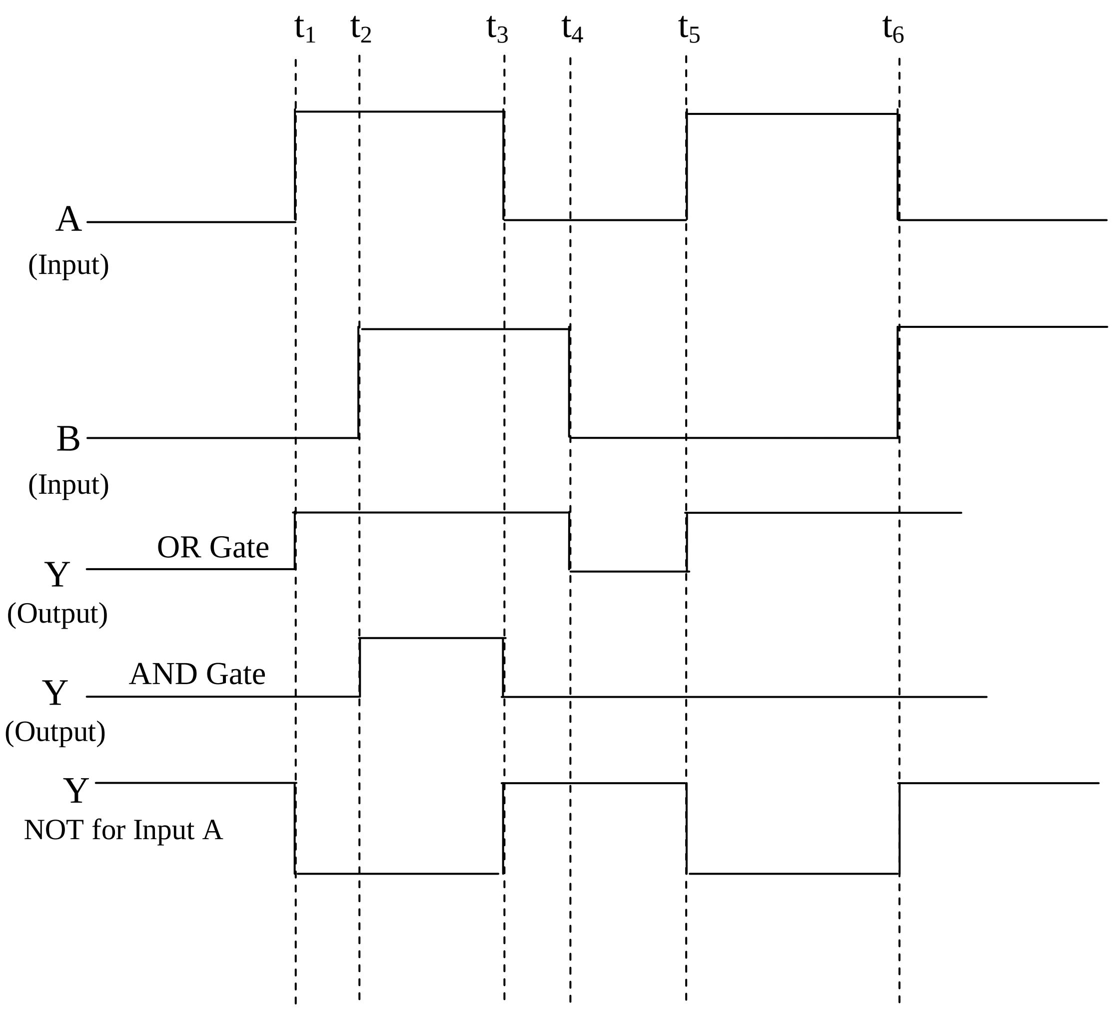
Problems:

1. Justify the output waveform (Y) of the OR gate, AND gate, NOT gate, NOR gate, and NAND gate for the following inputs A and B given in figure.

Solution Note the following: **<u>For OR gate:</u>**

- At $t < t_1$; A = 0, B = 0; Hence Y = 0
- For t_1 to t_2; A = 1, B = 0; Hence Y = 1
- For t_2 to t_3; A = 1, B = 1; Hence Y = 1
- For t_3 to t_4; A = 0, B = 1; Hence Y = 1
- For t_4 to t_5; A = 0, B = 0; Hence Y = 0
- For t_5 to t_6; A = 1, B = 0; Hence Y = 1
- For $t > t_6$; A = 0, B = 1; Hence Y = 1

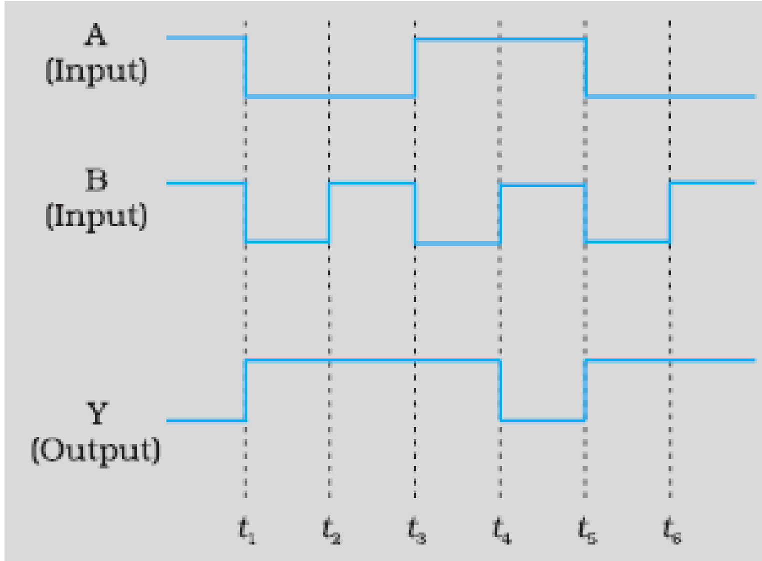
Therefore the waveform Y will be as shown in the figure.



2. Sketch the output Y from a NAND gate having inputs A and B given below:

Solution

- For t < t_1; A = 1, B = 1; Hence Y = 0
- For t_1 to t_2; A = 0, B = 0; Hence Y = 1
- For t_2 to t_3; A = 0, B = 1; Hence Y = 1
- For t_3 to t_4; A = 1, B = 0; Hence Y = 1
- For t_4 to t_5; A = 1, B = 1; Hence Y = 0
- For t_5 to t_6; A = 0, B = 0; Hence Y = 1
- For t > t_6; A = 0, B = 1; Hence Y = 1



Boolean algebra:

1.2.1 BASIC LAWS

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A + A = A$
4. $A + \overline{A} = 1$
5. $A . 0 = 0$
6. $A . 1 = A$
7. $A . A = A$
8. $A . \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + \overline{A}B = A + B$
11. $A + AB = A$

4. $y = A + \overline{B} + \overline{A}B + (A + B)\overline{A}\overline{B}$

$y = A + \overline{B} + \overline{A}B + (A + B)\overline{A}\overline{B}$

$= A + \overline{B} + \overline{A}B + A\overline{A}B + A\overline{B}B$

$= A + \overline{B} + \overline{A}B$ (As $A\overline{A}B = A\overline{B}B = 0$)

$= A + \overline{A}B + \overline{B}$ (Rearranging)

$= (A + \overline{A}B) + \overline{B}$

$= A + B + \overline{B}$ [as $(A + \overline{A}B) = (A + B)$]

$= A + (B + \overline{B})$

$= A + 1$ [as $(B + \overline{B}) = 1$]

$= 1$ [as $(A + 1) = 1$]

Simplification of Expression using Boolean Laws

1. $ABC + A\overline{B}C + AB\overline{C}$

$= AC(B + \overline{B}) + AB\overline{C}$

$= AC + AB\overline{C}$

$= A(C + B\overline{C}) = A(C + \overline{C})(B + C)$

$= A(B + C)$

2. $AB + \overline{A}B + \overline{A}\,\overline{B}$

$= (A + \overline{A})B + \overline{A}\,\overline{B}$

$= B + \overline{A}\,\overline{B}$

$= (B + \overline{A})\ (B + \overline{B}) = \overline{A} + B$

3. $y = A\overline{BC} + A\overline{B}\,\overline{C}D + \overline{C}D + AB\overline{C}$

$y = A\overline{BC} + A\overline{B}\,\overline{C}D + \overline{C}D + AB\overline{C}$

$= A\overline{BC}(1 + D) + \overline{C}D + AB\overline{C}$

$= AA\overline{BC} + \overline{C}D + AB\overline{C}$ [as $(1 + D) = 1$]

$= A\overline{BC} + AB\overline{C} + \overline{C}D$ (Rearranging the terms)

$= A\overline{C} + (\overline{B} + B) + \overline{C}D$

$= A\overline{C} + \overline{C}D$ (Rearranging the terms)

2.3.4. DE-MORGAN'S THEOREM

The De-Morgan's theorem is very much used for simplifying boolean equations. It contains two laws.

(a) *FIRST LAW*: The complement of sum of the variables is equal to the product of their complements.

$\overline{A + B} = \overline{A} . \overline{B}$

The logic diagram for the first law is shown in the fig-2.1

2.3.5. SIMPLIFICATION OF EXPRESSION USING BOOLEAN TECHNIQUES.

1) $Y = AB\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D}$

$= B\overline{C}\,\overline{D}(A + \overline{A}) + BC\overline{D}(A + \overline{A})$

$= B\overline{C}\,\overline{D} + BC\overline{D}$

$= B\overline{D} . (C + \overline{C})$

$Y = B\overline{D}$

2) $Y = AB + A(B+C) + B(B + C)$

(b) SECOND LAW . The complement of product of the variables is equal to the sum of their complements.

$$\overline{A.B} = \overline{A} + \overline{B}$$

The logic diagram for the second law is shown in the fig.2.2.

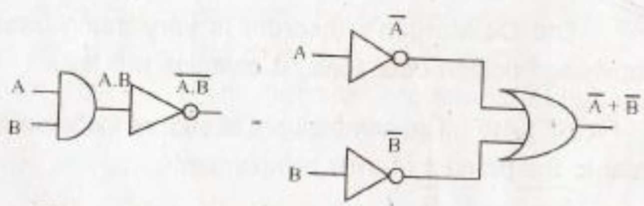


FIG 2.2 Logic diagram for De-morgan's second law

The truth table for verifying the De-morgans second law is shown in the table below.

A	B	A.B	$\overline{A.B}$	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1

Procedure for applying DeMorgan's Theorem

When using DeMorgan's theorems to reduce an expression, the following procedure is followed:

1. Break the inverter (NOT) sign at any point in the expression.
2. Change the operator sign at that point to its opposite. That is, + *(OR)* is changed to .*(AND)* and vice versa.0
3. Repeat steps 1 and 2 till expression is reduced to one in which only single variables are inverted.

DUALITY THEOREM

Example : 1

Consider the expression $A+0=A$. The dual relation is $A\cdot 1=A$. The dual property is obtained by changing the OR sign to an AND sign and by complementing the 0 to get a 1.

Example : 2

Consider the relation $A(B+C)=AB+AC$. The duality theorem produces a new Boolean relation $A+BC=(A+B)(A+C)$

Table. 3.2 shows some Boolean relations and their duals.

Boolean relation	Dual
$A+B=B+A$	$AB=BA$
$A+(B+C)=(A+B)+C$	$A(BC)=(AB)C$
$A(B+C)=AB+AC$	$A+(BC)=(A+B)(A+C)$
$A+0=A$	$A\cdot 1=A$
$A+1=1$	$A\cdot 0=0$
$A+A=A$	$A\cdot A=A$
$A+\overline{A}=1$	$A\cdot\overline{A}=0$
$\overline{\overline{A}}=A$	$\overline{\overline{A}}=A$
$\overline{A+B}=\overline{A}\,\overline{B}$	$\overline{AB}=\overline{A}+\overline{B}$
$A+AB=A$	$A(A+B)=A$
$A+\overline{A}B=A+B$	$A(\overline{A}+B)=AB$

Table. 3.2 : Boolean relations and duals

SUMMARY

All the laws, rules and theorems of Boolean algebra are summarized in Table. 3.3 for ready reference.

Laws of Boolean Algebra	
Commutative law of addition	$A+B=B+A$
Commutative law of multiplication	$A\cdot B=B\cdot A$
Associative law of addition	$A+(B+C)=(A+B)+C$
Associative law of multiplication	$A(BC)=(AB)C$
Distributive law	$A(B+C)=AB+AC$
Rules of Boolean Algebra	
Rule 1	$A+0=A$
Rule 2	$A+1=1$

REPRESENTATION OF EXPRESSIONS

The following two forms of Boolean expressions are commonly used:

1. Sum-of-products form
2. Product-of-sums form

SUM-OF-PRODUCTS (SOP) FORM

A product of two or more variables in Boolean algebra is the AND function and a sum is the OR function. Therefore, a sum-of-products expression is two or more AND functions ORed together. It can also contain a term with a single variable.

Examples:

Some examples of expressions in sum-of-products form are as follows:

$A\bar{B}C + DEF$

$A + BCD + DEF$

PRODUCT-OF-SUMS (POS) FORM

The product-of-sums form is the dual of the sum-of-products. In terms of logic functions, it is two or more OR functions ANDed together. It can also contain a term with a single variable.

Examples:

Some examples of expressions in product-of-sums form are as follows:

$(A + \bar{B} + C)(D + E + F)$

$A(B + C + D)(D + E + F)$

SIMPLIFICATION OF LOGIC EXPRESSIONS USING BOOLEAN ALGEBRA

Every Boolean expression must be simplified before realization because it reduces the cost and complexity of the digital hardware and increases its reliability. To reduce Boolean expressions, all the laws of Boolean algebra may be used as follows:

1. Multiply all variables necessary to remove parentheses.
2. Look for identical terms. Retain only one of those terms and drop all others. For example,

   $AB + AB + AB + AB = AB$

   $AB \cdot AB \cdot AB \cdot AB = AB$
3. Look for pairs of variable and its negation in the same term. This term can be dropped. For example,

   $A \cdot B\bar{B} = A \cdot 0 = 0$

SOLVED PROBLEMS

Problem 1 : Simplify the expression $y = A\overline{B}D + A\overline{B}\,\overline{D}$

Solution:

Step 1: Take $A\overline{B}$ common :

$$y = A\overline{B}(D + \overline{D})$$

Step 2: Apply Rule 6 $(D + \overline{D}) = 1$:

$$y = A\overline{B} \cdot 1$$

Step 3: Apply Rule 4 ($A\overline{B} \cdot 1 = A\overline{B}$):

$$y = A\overline{B}$$

Problem 2 : Simplify $z = (\overline{A} + B)(A + B)$

Solution:

Step 1: Expand the expression by multiplying out the terms:

$$z = \overline{A}A + \overline{A}B + BA + BB$$

Step 2: Apply Rule 8 ($\overline{A}A = 0$) and Rule 7 ($BB = B$):

$$z = 0 + \overline{A}B + AB + B = \overline{A}B + AB + B$$

Step 3: Take B common:

$$z = B(\overline{A} + A + 1)$$

Step 4: Apply Rule 2 ($\overline{A} + A + 1 = 1$) :

$$z = B \cdot 1$$

Step 5: Apply Rule 4 ($B \cdot 1 = B$):

$$z = B$$

Problem 3 : Simplify $x = ACD + \overline{A}BCD$

Solution:

Step 1: Take CD common:

$$x = CD(A + \overline{A}B)$$

Step 2: Apply Rule 11 $(A + \overline{A}B = A + B)$

$$x = CD(A + B)$$