Dielectric Properties of Matter

Syllabus _Dielectric Properties of Matter: Electric Field in matter, Polarization, Polarization charge, Electrical Susceptibility and Dielectric constant, Capacitor (Parallel Plate, Spherical, Cylindrical) field with dielectric, Displacement Vector D, Relation between E, P and D, Gauss law in dielectrics.

Dielectrics:

Dielectrics are insulating materials in which all the electrons are tightly bound to their parent molecules (nucleus) and there are no free electrons to carry current. Hence the electrical conductivity of a dielectric is very low. The conductivity of an ideal dielectric is zero. Even with normal voltage or thermal energy electrons are not released. Dielectrics are non metallic materials of high specific resistance and have negative temperature coefficient of resistance. When a dielectric is placed in an external electric field, induced charges are produced on its outer surface.

Examples: Glass, Plastic, Mica, Oil, Wax, Air etc.

Polar and Non-Polar molecules in electric field: (Dielectric placed in electric field)

Insulators or dielectrics are substances which do not have free charges (electrons) like the conductors of electrical charge i.e. dielectric substances are neutral in spite of charge contained in them. This is due to the reason that they contain equal and opposite charges. Positive charge (in the form of proton) is contained within the nucleus while the negative charge (in the form of electrons) is distributed all around it. A centre of charge is defined as a point where whole of the positive or negative charge may be supposed (assumed) to be concentrated. Thus, there are two centres of charges; one for positive and the second for negative charge. Depending on the positions of these charge centers, the dielectric medium is divided into two categories.

Polar molecules:

If the centre of charges for positive and negative charges do not coincide then the molecule is equivalent to an electric dipole and is said to be a polar molecule. Such a molecule is associated with an electric dipole moment. When subjected to an electric field, the molecule experiences a torque and tends to fall in line with the direction of lines of force of the electric field. Examples: H_2O, CO, N_2O, NH_3, HCl are the polar molecules.

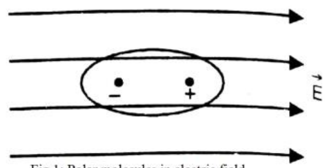


Fig.1: Polar molecules in electric field

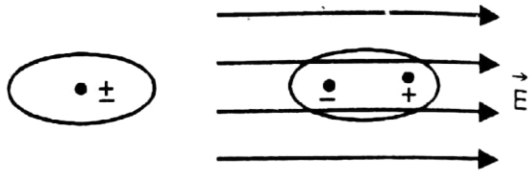


Fig.2: Non-Polar molecules in electric field.

Non-Polar molecules:

If the centres of charges for positive and negative charges coincide at a point then the molecule are said to be a non-polar molecule. Such a molecule does not have a permanent electric dipole moment. When subjected to an electric field, there will be some relative displacement between the positive and negative charges. As a result, the centres of charges for positive and negative charges will not remain coincide. Thus a non-polar molecule when subjected to an electric field acquires an electric dipole moment.

Thus, polar and non-polar molecules behave in a similar manner when subjected to an electric field.

Examples: H_2, O_2, N_2, CO_2, He, Ne, are the non-polar molecules.

Action of the dielectric:

A dielectric medium is always neutral whether it is made up of polar or non-polar molecules. The molecules are arranged at random. In case of polar molecules, the end of one having positive charge lies near the end of the other having negative charge (fig.a), thus neutralizing each other effect. Let this medium be placed in between the two plates P and Q of a capacitor (fig.b). The electric field 'E' in between the plates exerts a torque on each of the dipole and tends to bring it along its line of force. Greater the strength of the field greater is the torque and hence more is the number of dipoles which get aligned along the lines of force. Fig.b shows that all the dipoles have aligned themselves along the lines of force. In this stage the medium is said to be polarized. It will be observed that the two opposite of faces of the dielectric acquire positive and negative charges (Fig.c) and the medium is said to be polarized. This is due to the reason that the charges in between two dotted lines AB and CD neutralized each other's. Hence net charge within the dielectric is zero except near the surfaces of the dielectric.

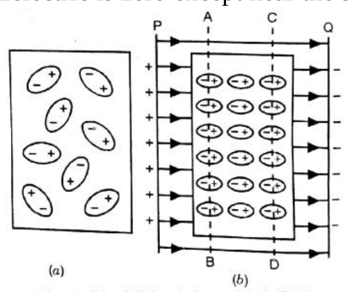
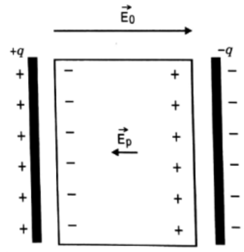


Fig. A slab of dielectric in an electric field.



(C)

The appearance of the net negative and positive charges on opposite faces of the dielectric produces an electric field $\vec{E}_p$ within the dielectric. This field acts in the opposite of the applied field $\vec{E}_0$. Therefore the electric field inside the dielectric is reduced. So net electric field inside the dielectric will be

$$\vec{E} = \vec{E}_0 - \vec{E}_p$$

Hence, when a dielectric slab is placed in an electric field, the strength of electric field will decrease. *The phenomenon of appearance of charges on opposite faces of a dielectric slab, induced by external electric field is called polarization.*

The charge on the surface of plates of a parallel plate capacitor is called free charge. The surface density due to these free charges is called surface density of free charges σ_f .

The induced charge on the surface of a polarised dielectric is called bound charge. The surface density due to bound charge is called surface density of bound charge σ_p or surface density of induced charge σ_i .

Fundamental definitions in dielectrics:

Polarization vector ($\vec{P}$):

The measurement of polarization of dielectric is done by polarization vector. The induced electric dipole moment per unit volume of a dielectric when placed in an electric field is called Polarization vector ($\vec{P}$). This vector also is sometimes called as polarization density.

Let us consider a dielectric slab of thickness d , surface area A and volume V in an electric field. If $\vec{p_i}$ is the induced dipole moment of the dielectric then the polarization vector

$$\vec{P} = \frac{\vec{p}_i}{V}$$

$$\Rightarrow \vec{P} = \frac{q_i \vec{d}}{V}$$

The direction of $\vec{P}$ is from $-q_i \quad to \quad q_i$.

Magnitude of polarization vector is

$$P = \frac{q_i d}{V} = \frac{q_i d}{Ad} = \frac{q_i}{A} = \sigma_p$$

Hence polarization of a dielectric is numerically equal to surface density of bound charge or surface density of induced charge. The SI unit of P is coulomb/meter^2.

Susceptibility (χ_e):

When a dielectric material is placed in an electric field, it becomes electrically polarised and the polarization vector $\vec{P}$ is proportional to the electric field $\vec{E}$ inside the dielectric i.e.

$$\vec{P} \propto \vec{E}$$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

Where, χ_e is called electric susceptibility and ε_0 is the permittivity of air or vacuum. In vacuum there is no polarization. Hence susceptibility for vacuum is zero. χ_e has no units in SI system.

Dielectric constant ($K \; Or \; \varepsilon_r$):

The dielectric characteristics of a material are determined by the dielectric constant or relative permittivity ε_r of that material. The ratio of the electric field inside the capacitor without dielectric to the electric field inside the capacitor with dielectric is called dielectric constant. It is denoted by $K \; Or \; \varepsilon_r$.

Let us consider a parallel plate capacitor, having a uniform electric field $\vec{E}_0$. If $\vec{E}_p$ is the electric field produced inside the dielectric due to induced charges then net electric field inside the dielectric is

$$\vec{E} = \vec{E}_0 - \vec{E}_p$$

Thus dielectric constant,

$$K = \frac{E_0}{E}$$

$$\Rightarrow K = \frac{E_0}{E_0 - E_p} \qquad \text{---- (1)}$$

As, $E_0 > E_p \quad \therefore \quad K > 1$

If σ is the surface charge density due to free charges on plate of capacitor then electric field between the plates of capacitor with air or vacuum as medium is

$$E_0 = \frac{\sigma}{\varepsilon_0} \qquad \text{----- (2)}$$

Again, if σ_p is the surface charge density due to induced (bound) charges inside the dielectric then electric field due to induced charges inside the dielectric is

$$E_p = \frac{\sigma_p}{\varepsilon_0} \qquad \text{---- (3)}$$

Putting the values from equations (2) and (3) in equation (1), we get

$$K = \frac{\frac{\sigma}{\varepsilon_0}}{\frac{\sigma}{\varepsilon_0} - \frac{\sigma_p}{\varepsilon_0}}$$

$$\Rightarrow K = \frac{\sigma}{\sigma - \sigma_p} \qquad \text{---- (4)}$$

Special Cases:

1. If the dielectric sample is of metal then $\sigma_p = \sigma$. Therefore, $K = \infty$. It means metal has infinite dielectric constant.
2. If the dielectric is vacuum then $\sigma_p = 0$. Therefore, $K = 1$. Thus dielectric constant of vacuum is one.

Let C_0 be the capacity of a capacitor without dielectric (i.e. with vacuum or air as medium) and V_0 be the potential difference between the plates separated by a distance d .

Then, $$C_0 = \frac{q}{V_0} = \frac{q}{E_0 d} \qquad \text{---- (5)}$$

Again, let C be the capacity of a capacitor with dielectric and V be the potential difference between the plates separated by a distance d .

Then, $$C = \frac{q}{V} = \frac{q}{Ed} \qquad \text{---- (6)}$$

Therefore, $$\frac{C}{C_0} = \frac{\frac{q}{Ed}}{\frac{q}{E_0 d}} = \frac{E_0}{E} \qquad \text{---- (7)}$$

Since, $E_0 > E \quad \therefore \quad C > C_0$

Hence, capacity of a capacitor is increases on introducing a dielectric slab between the plates.

Now, $$\frac{V_0}{V} = \frac{E_0 d}{Ed} = \frac{E_0}{E} \qquad \text{---- (8)}$$

Since, $E_0 > E \quad \therefore \quad V_0 > V$

Hence, potential of a capacitor is decreases on introducing a dielectric slab between the plates.

Dielectric constant (ε_r): (Another Definition)

The dielectric characteristics of a material are determined by the dielectric constant or relative permittivity ε_r of that material. Dielectric constant is the ratio between the permittivity of the medium ε_m and the permittivity of free space ε_0 i.e.

$$\varepsilon_r = \frac{\varepsilon_m}{\varepsilon_0}$$

Since, it is a ratio of same quantity, ε_r has no unit. It is a measure of polarization in the dielectric material.
Since, $E < E_0,\ \ \varepsilon_m > \varepsilon_0,$ so that $\varepsilon_r > 1$. Thus we can write

$$\varepsilon_r = 1 + \chi_e$$

Where, χ_e is called electrical susceptibility.

Relation between Dielectric constant (K) and Susceptibility (χ_e):

Let us consider a parallel plate capacitor with air or vacuum as medium between the plates of the capacitor. If q be the charge on the capacitor and σ be the surface charge density on the plates of capacitor then, uniform electric field between the plates of capacitor is

$$E_0 = \frac{\sigma}{\varepsilon_0} \qquad \text{---- (1)}$$

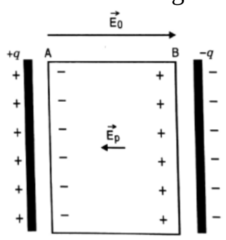
Now introduce a dielectric slab between the plates of the capacitor. The dielectric gets polarised and net electric field inside the dielectric is

$$E = E_0 - E_p \qquad \text{---- (2)}$$

Where, E_p is the electric field produced inside the dielectric due to induced charges or bound charges as shown in figure and is given by

$$E_p = \frac{\sigma_p}{\varepsilon_0} \qquad \text{---- (3)}$$

Where, σ_p is the surface charge density due to induced charges produced inside the dielectric.



Putting this value in equation (2), we get

$$E = E_0 - \frac{\sigma_p}{\varepsilon_0} \qquad \text{---- (4)}$$

We know that polarization vector,

$$P = \sigma_p$$

Therefore, $E = E_0 - \frac{P}{\varepsilon_0}$

Dividing both sides by E, we get

$$\frac{E}{E} = \frac{E_0}{E} - \frac{P}{\varepsilon_0 E}$$

Since, dielectric constant $K = \frac{E_0}{E}$, add electric susceptibility $\chi_e = \frac{P}{\varepsilon_0 E}$

$$\Rightarrow 1 = K - \chi_e$$

$$\Rightarrow K = 1 + \chi_e$$

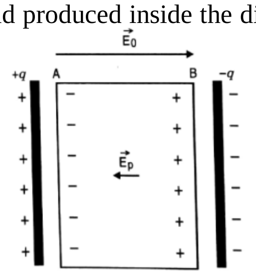
This is the relation between dielectric constant and electric susceptibility.

Electric Displacement Vector $\vec{D}$: (Relation between $\vec{E}$, $\vec{P}$ and $\vec{D}$)

Let us consider a parallel plate capacitor with air or vacuum as medium between the plates of the capacitor. If q be the charge on the capacitor and σ_{free} be the surface charge density of free charges on the plates of capacitor then, uniform electric field between the plates of capacitor is

$$E_0 = \frac{\sigma_{free}}{\varepsilon_0} \qquad \text{---- (1)}$$

When a dielectric slab is introduce between the plates of the capacitor then the dielectric gets polarised. If σ_p is the surface charge density due to induced charges (bound charges) produced inside the dielectric then the electric field produced inside the dielectric is



$$E_p = \frac{\sigma_p}{\varepsilon_0} \qquad \text{---- (2)}$$

Now, net electric field within the dielectric is

$$E = E_0 - E_p$$

$$\Rightarrow E_0 = E + E_p$$

$$\Rightarrow \frac{\sigma_{free}}{\varepsilon_0} = E + \frac{\sigma_p}{\varepsilon_0}$$

$$\Rightarrow \frac{\sigma_{free}}{\varepsilon_0} = E + \frac{P}{\varepsilon_0} \qquad [\because P = \sigma_p]$$

$$\Rightarrow \frac{\sigma_{free}}{\varepsilon_0} = \frac{\varepsilon_0 E + P}{\varepsilon_0}$$

$$\Rightarrow \sigma_{free} = \varepsilon_0 E + P$$

$$\Rightarrow D = \varepsilon_0 E + P \qquad \text{----- (3)}$$

The quantity $\varepsilon_0 E + P$ is of special significance called electric displacement vector $\vec{D}$. In vector form

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \qquad \text{----- (4)}$$

The direction of $\vec{D}$ is same as $\vec{E}$ and $\vec{P}$ .This is the relation between $\vec{E}$, $\vec{P}$ and $\vec{D}$, where

$$\vec{D} = \frac{q}{A} = \sigma_{free}$$

The units of $\vec{D}$ and $\vec{P}$ is $Coulomb/meter^2$.

Capacitance of a parallel plate Capacitor partly filled with a Dielectric Slab:

Let us consider two parallel metals plates P & S each of area 'A' and separated by a distance 'd' and let the medium between the plates is air. Let a charge +q is given to the plate P so that the surface charge density on it is σ .
Electric field intensity at any point in the air space between the plates is

$$E_0 = \frac{\sigma}{\varepsilon_0} \qquad \text{--- (1)}$$

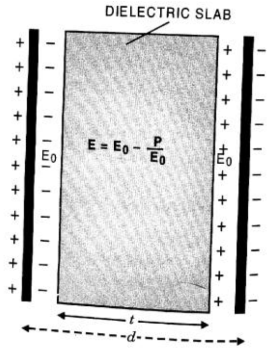
Let a dielectric slab of thickness t and relative permittivity K or ε_r is introduced between the plates. The molecules of dielectric slab get polarised and induced charges are produced on the dielectric slab and an induced electric field E_p is produced inside the dielectric due to these induced charges.

Therefore, net electric field inside the dielectric is

$$E = E_0 - E_p$$

The potential difference V between the plates is the work done in carrying a unit positive charge from one plate to other in the electric field E_0 over a length $(d-t)$ and in the field E over a length 't'. Thus

$$V = E_0(d-t) + Et \qquad \text{---- (2)}$$



We know that dielectric constant,

$$K = \frac{E_0}{E} \Rightarrow E = \frac{E_0}{K}$$

Putting this value in equation (2), we get

$$V = E_0(d-t) + \frac{E_0}{K}t$$

$$\Rightarrow V = E_0\left((d-t) + \frac{t}{K}\right)$$

$$\Rightarrow V = \frac{\sigma}{\varepsilon_0}\left((d-t) + \frac{t}{K}\right)$$

$$\Rightarrow V = \frac{q}{\varepsilon_0 A}\left((d-t) + \frac{t}{K}\right) \qquad \text{---- (3)}$$

Hence, the capacitance of the capacitor is

$$C = \frac{q}{V} = \frac{q}{\frac{q}{\varepsilon_0 A}\left\{(d-t) + \frac{t}{K}\right\}} = \frac{\varepsilon_0 A}{\left\{(d-t) + \frac{t}{K}\right\}} \qquad \text{---- (4)}$$

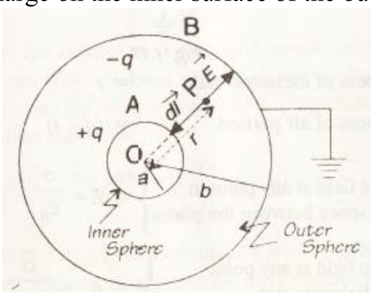
If the space between two plates of the capacitor is completely filled with a dielectric of dielectric constant K then $t = d$ and the capacitance of the capacitor become

$$C_m = \frac{\varepsilon_0 A}{\frac{d}{K}} = K\frac{\varepsilon_0 A}{d} = KC_{air}$$

Since, $K > 1$, hence capacitance of a capacitor increases after introducing a dielectric slab between the plates.

Capacitance of a Spherical Capacitor:

Let A and B be two concentric metal spheres of radius 'a' and 'b' respectively with air as the intervening medium. The outer sphere B is earthed. A charge '+q' is given to the inner sphere. The induced charge on the inner surface of the outer sphere is '-q'. ler 'P' is a point at a distance 'r' from the common centre 'O'.



Now, electric field at the point P is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{----- (1)}$$

Where $\hat{r}$ is the unit vector along $\vec{OP}$.

The potential difference between the spheres A and B is given by

$$V = -\int_b^a \vec{E}\bullet\vec{dl} = -\int_b^a Edl\cos 180^0 = \int_b^a Edl \qquad \text{----- (2)}$$

Here, $\vec{dl}$ is the differential vector displacement along a path from B to A. Further, in moving a distance dl in the direction of motion, we are moving in the direction of r decreasing, so that $dl = -dr$. Hence, from equation (1) and (2), we have

$$V = -\int_b^a Edr = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_b^a = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right) \qquad \text{--- (3)}$$

Therefore, Capacitance of the spherical capacitor is

$$C = \frac{q}{V} = q \times \frac{4\pi\varepsilon_0}{q}\frac{ab}{(b-a)}$$

$$\Rightarrow C = 4\pi\varepsilon_0\frac{ab}{(b-a)} \qquad \text{---- (4)}$$

Equation (4) can be written in the form

$$C = 4\pi\varepsilon_0\frac{ab}{(b-a)} = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$$

When $b \to \infty$, $C = 4\pi\varepsilon_0 a$

This is the capacitance of an isolated conducting sphere of radius 'a'.

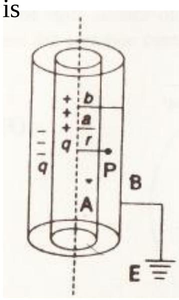
If the space between two spheres is completely filled with a dielectric of dielectric constant K then the capacitance of the spherical capacitor is

$$C_m = 4\pi\varepsilon_0 K \frac{ab}{(b-a)}$$

Capacitance of a Cylindrical Capacitor:

Let us consider a cylindrical capacitor of length 'l' formed by two coaxial cylinder A and B of radius 'a' and 'b' respectively with air as the intervening medium. The outer cylinder B is earthed. If a charge '+q' is given to the inner cylinder then an equal charge –q is induced on the inner surface of the outer cylinder.

The electric field at a point P in the space between the two cylinders at a distance 'r' from the axis is



$$\vec{E} = \frac{1}{2\pi\varepsilon_0 l}\frac{q}{r}\hat{r} \qquad \text{----- (1)}$$

Where $\hat{r}$ is the unit vector along $\vec{OP}$.

The potential difference between the cylinder A and B is given by

$$V = -\int_b^a \vec{E}\bullet\vec{dl} = -\int_b^a Edl\cos 180^0 = \int_b^a Edl \qquad \text{----- (2)}$$

Here, $\vec{dl}$ is the differential vector displacement along a path from B to A. Further, in moving a distance dl in the direction of motion, we are moving in the direction of r decreasing, so that $dl = -dr$. Hence, from equation (1) and (2), we have

$$V = -\int_b^a Edr = -\frac{q}{2\pi\varepsilon_0 l}\int_b^a \frac{dr}{r} = -\frac{q}{2\pi\varepsilon_0 l}[\log r]_b^a = -\frac{q}{2\pi\varepsilon_0 l}[\log a - \log b] = \frac{q}{2\pi\varepsilon_0 l}\log\left(\frac{b}{a}\right) \qquad \text{--- (3)}$$

Therefore, Capacitance of the cylindrical capacitor is

$$C = \frac{q}{V} = q \times \frac{2\pi\varepsilon_0 l}{q\log\left(\frac{b}{a}\right)}$$

$$C = \frac{2\pi\varepsilon_0 l}{\log\left(\frac{b}{a}\right)} \qquad \text{---- (4)}$$

If the space between the two cylinders contains some dielectric medium of relative permittivity or dielectric constant K, then the above expression becomes

$$C = \frac{2\pi\varepsilon_0 K l}{\log\left(\frac{b}{a}\right)}$$

Examples of practical cylindrical capacitors:

1. The Co-axial cable consists of a cylindrical metal shield, a co-axial central conductor and an interposed dielectric.
2. A submarine cable consists of strands of copper separated from the surrounding water by a suitable insulating cable. It thus acts as a cylindrical capacitor. The copper strands form the inner cylinder. The surrounding water acts as the outer cylinder. The insulating casing acts as the dielectric.

Gauss's Law in Dielectric:

It states that the total electric flux over a closed surface in a dielectric is equal to $\frac{1}{\varepsilon_0 K}$ or $\frac{1}{\varepsilon}$ times the free charge q enclosed by the surface i.e. mathematically

$$\oint\!\!\oint_S \vec{E}\bullet\vec{dS} = \frac{q}{\varepsilon_0 K} = \frac{q}{\varepsilon}$$

Here, ε_0 is the permittivity of free space, K is the dielectric constant and $\varepsilon = \varepsilon_0 K$ is the permittivity of the medium.

Proof:

Let us consider a parallel plate capacitor of plate's area A with air or vacuum as medium between the plates of the capacitor. Let $\vec{E_0}$ is the electric field between the plates of the capacitor. If PQRS be the Gaussian surface as shown in figure: 1, then electric flux through the Gaussian surface with vacuum as medium between the plates of the capacitor according to Gauss's law is

$$\oint\!\!\oint_S \vec{E_0}\bullet\vec{dS} = \frac{q}{\varepsilon_0} \qquad \text{---- (1)}$$

Where, q is the charge enclosed by Gaussian surface and $\vec{dS}$ is surface element of area.

Since, $\vec{E_0}$ and $\vec{dS}$ are parallel, therefore from equation (1), we have

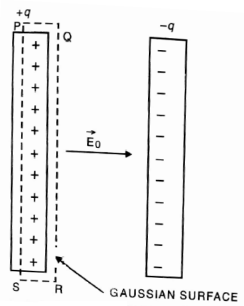


Fig.1

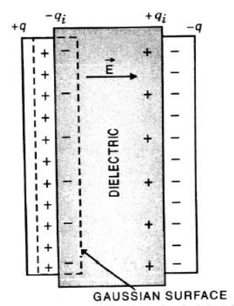


Fig.2

$$\oiint_S E_0 dS\cos 0 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E_0 \oiint_S dS = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E_0 A = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E_0 = \frac{q}{\varepsilon_0 A} \qquad \text{----- (2)}$$

Let us now introduce a dielectric material of dielectric constant K between the plates of the capacitor as shown in figure: 2. Then induced charges appear on the surfaces of dielectric. These induced charges produce their own electric field which opposes the external electric field $\vec{E_0}$. Let $\vec{E}$ is the resultant electric field within the dielectric. If q_i is the induced surface charge then charge within the Gaussian surface will be $(q-q_i)$.

Now the Gauss's law will be

$$\oiint_S \vec{E}\bullet\vec{dS} = \frac{q-q_i}{\varepsilon_0} \qquad \text{--- (3)}$$

$$\Rightarrow \oiint_S EdS\cos 0 = \frac{q-q_i}{\varepsilon_0}$$

$$\Rightarrow E\oiint_S dS = \frac{q-q_i}{\varepsilon_0}$$

$$\Rightarrow EA = \frac{q-q_i}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q-q_i}{\varepsilon_0 A} \qquad \text{--- (4)}$$

Since, according to definition of dielectric constant

$$K = \frac{E_0}{E} \Rightarrow E = \frac{E_0}{K} \qquad \text{--- (5)}$$

From equation (4) and (5),

$$\frac{E_0}{K} = \frac{q-q_i}{\varepsilon_0 A}$$

$$\Rightarrow \frac{q}{\varepsilon_0 AK} = \frac{q-q_i}{\varepsilon_0 A} \qquad \text{(Using equation.2)}$$

$$\Rightarrow \frac{q}{K} = q-q_i$$

$$\Rightarrow q_i = q-\frac{q}{K} = q\left(1-\frac{1}{K}\right)$$

Putting this value in equation (3), we get

$$\oiint_S \vec{E}\bullet\vec{dS} = \frac{q}{\varepsilon_0}-\frac{q_i}{\varepsilon_0}$$

$$\Rightarrow \oiint_S \vec{E}\bullet\vec{dS} = \frac{q}{\varepsilon_0}-\frac{q\left(1-\frac{1}{K}\right)}{\varepsilon_0}$$

$$\Rightarrow \oiint_S \vec{E}\bullet\vec{dS} = \frac{q}{\varepsilon_0}\left(1-1+\frac{1}{K}\right)$$

$$\therefore \oiint_S \vec{E}\bullet\vec{dS} = \frac{q}{\varepsilon_0 K}$$

This is the Gauss's Law in dielectric.